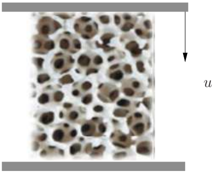
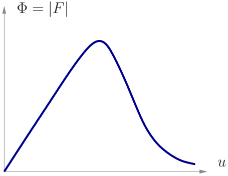
Physical response of random networks generated from the same random graph





$$\mathbf{\Phi} = f_{\Phi}(\ldots, \underbrace{\gamma_1, \ldots, \gamma_L}_{\text{geometry}}, \underbrace{\theta_1, \ldots, \theta_K}_{\text{topology}})$$

Uwe Mühlich

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   - time and resource consuming
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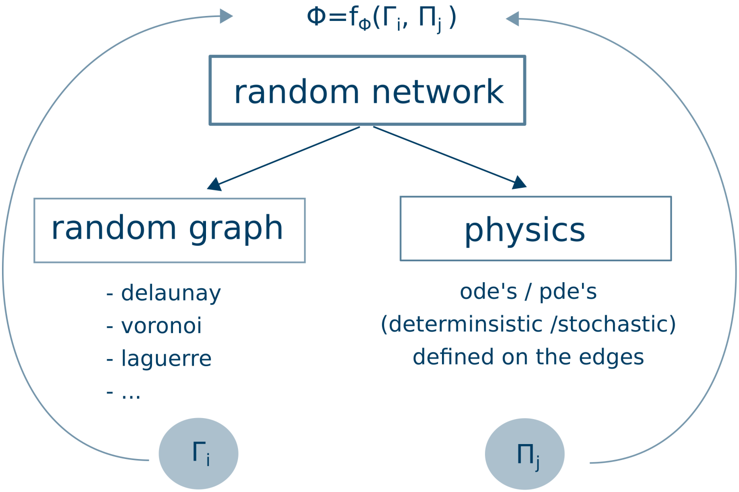
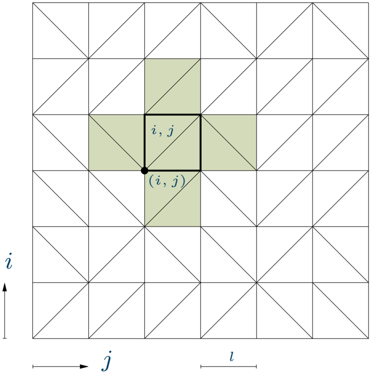
3. Define and study rigorously simplified problems first.
   - investigation methodology
   - eventually a smarter version of option 2



$\Phi = f_{\Phi}(\Gamma_i, \Pi_j)$
random network
random graph
- delaunay
- voronoi
- laguerre
- ...
physics
ode's / pde's
(determinsistic /stochastic)
defined on the edges
Γ_i
Π_j

Parent Graph
(0 graph)



discrete field ξ on a lattice:

$$\xi_{i,j} = \begin{cases} 0 & \text{if diagonal} \quad / \\ 1 & \quad\quad\quad\quad\quad\ \ \backslash \end{cases}$$

ratio between / and \ diagonals $\rightarrow \rho$:

$$\rho = \frac{1}{M^2} \sum_{i,j} \xi_{i,j}$$

$$\bar{\rho} = \min(\rho, 1-\rho) \quad \in [0, 0.5]$$

nearest neighbor correlation $\rightarrow \mu$:

$$\mu = \frac{1}{M^2} \sum_{i,j} \sum_{\substack{k,l \\ d=1}} \mathbb{1}(\xi_{i,j} \neq \xi_{k,l})$$

with $d = |k-i| + |l-j|$

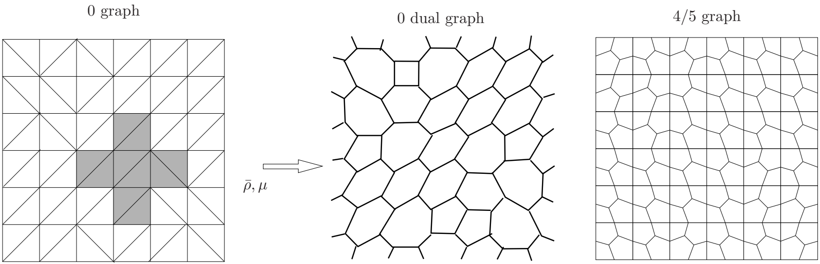
Sampling procedure

- set of all possible configurations $\mathcal{C} = \{X_1, X_2, ..., X_{2^{M^2}}\}$
- probability of finding a configuration $X_k \in \mathcal{C}$

$$P(X = X_k) = \frac{1}{\mathcal{Z}} \exp\left(-\beta\, \mu(X_k)\right)$$

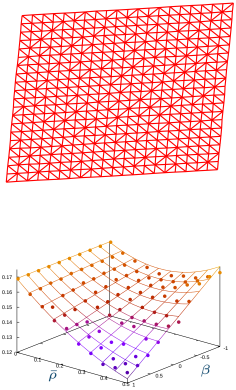
- $\mathcal{Z}$: partition function which ensures that $\sum_{i=1}^{2^{M^2}} P(X = X_i) = 1$
- β: scalar parameter which controls the sampling procedure
- sampling according to $P(X)$ for fixed $\bar{\rho}$ using Metropolis
- bijective relation $\mu \leftrightarrow \beta$

Derived graphs

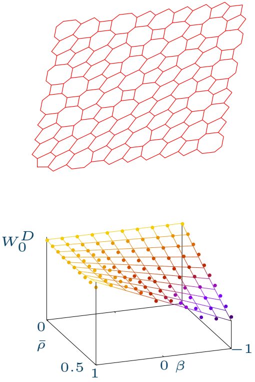


Networks: physics → deformation, response → strain energy

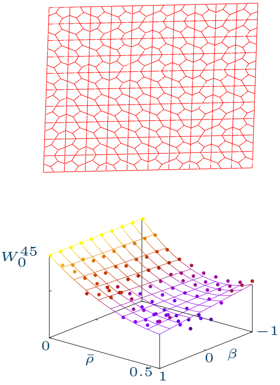
truss - 0



beam - 0 dual

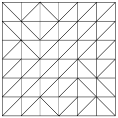


beam - 45

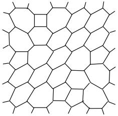


General measures in the absence of a 0-graph

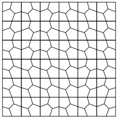
0 graph



0 dual



4/5 graph



measure	symbol	0 graph		0 dual		4/5 graph	
		mean	var	mean	var	mean	var
node degree	n_d	6	✓	3	0	3.5	0.25
cell degree	c_d	3	0	6	?	4.5	0.25
sidedness (nearest nbs.)	s_R	3	0	?	?	?	?
cell area	A_C	$A_\triangle$	0^*	$1^\dagger$	?	$\frac{A_4+A_5}{2}$	$\frac{(A_4-A_5)^2}{2}$
area moments	I_{11} I_{22} I_{12}						
edge angle $^+$	α_e	✓					

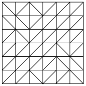
*) without varying node coordinates †) for dimensions used here

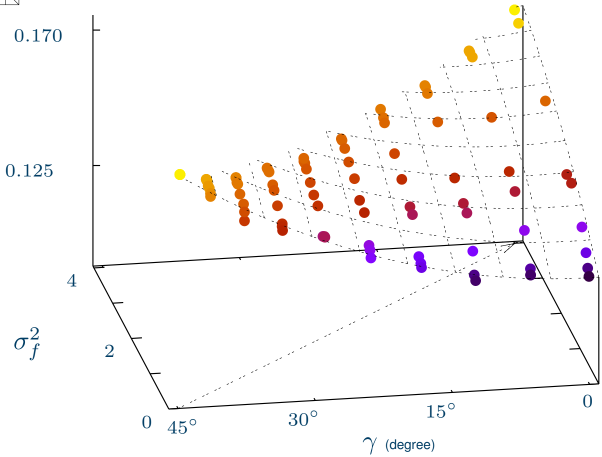
+) $\tan\alpha_e = \frac{\sum l_e \sin\alpha_e}{\sum l_e \cos\alpha_e}$

trial for 0 dual and 4/5 graph: s_R and I_{12}

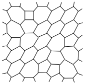
Results

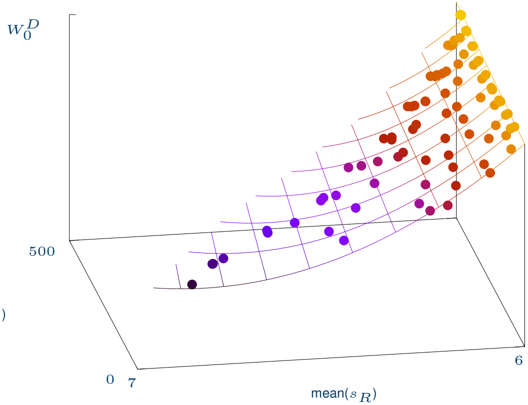
truss - 0



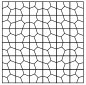


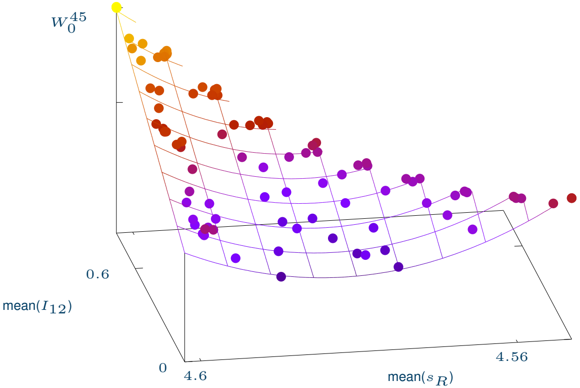
beam - 0 dual





beam - 4/5





Summary

- networks derived from 0 graph:

$$\Phi = f_\Phi(\mathbb{E}, \bar{\rho}, \mu) \Rightarrow \begin{array}{ll} f_\Phi^{\triangle}(\mathbb{E}, \text{var}(n_d), \alpha) & \text{ode 2} \\ f_\Phi^{\diamond}(\mathbb{E}, \text{mean}(s_d), \text{mean}(I_{12})) & \text{ode 4} \end{array}$$

- so far: 1 topological measure and 1 geometrical measure
- suspiciously simple - most likely because physics rather simple
  - linear
  - $\mathbb{E}$ (e.g. no overall bending)
  - time independent

  to be traced back to parent graph

$$\bar{\rho}, \mu \qquad \text{complexity} \uparrow \qquad \bar{\rho}, \mu, ...?$$

- reasonable methodology for performing numerical experiments

Outlook

- analytical estimates
  - e.g., mean field

$$\underline{\underline{\boldsymbol{K}}} \cdot \underline{\boldsymbol{u}} = \underline{\boldsymbol{b}} \qquad \longrightarrow \qquad \underline{\underline{\boldsymbol{K}}}^* \cdot \langle \underline{\boldsymbol{u}} \rangle = \underline{\boldsymbol{b}} \quad \text{with} \quad \underline{\underline{\boldsymbol{K}}}^* = \left\langle \underline{\underline{\boldsymbol{K}}} \right\rangle + \underline{\underline{\boldsymbol{K}}}_0^{-1} \mathrm{var}(\underline{\underline{\boldsymbol{K}}})$$

  - linear transport (work in progress)

$$\underline{\underline{\boldsymbol{K}}} = \underline{\underline{\boldsymbol{D}}} - \underline{\underline{\boldsymbol{A}}} \quad , \quad n_d, s_d \rightarrow \left\langle \underline{\underline{\boldsymbol{D}}} \right\rangle, \left\langle \underline{\underline{\boldsymbol{A}}} \right\rangle, \text{var}(\underline{\underline{\boldsymbol{D}}}), \ldots$$

- physics:
  - linear / nonlinear transport
  - fracture
  - dynamics
- more general cases

Delaunay	$\longrightarrow$	var(n_d), α_e
Voronoi / Laguerre		mean(s_R), mean(I_{12})